Dynamical critical exponent of a nonequilibrium Ising model

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The dynamical critical exponent z of the two-dimensional Ising model with competing Glauber and Kawasaki processes is evaluated by Monte Carlo simulations. This model exhibits the phenomenon of self-organization when the Kawasaki dynamics is the dominant one. We have calculated the exponent z as a function of the parameter that controls the competition between the two processes. Our calculations are performed at the phase boundary that separates the nonequilibrium ordered and disordered phases. The values of z we obtain are around $z=2$ for all values of the competing parameter. When the Glauber dynamics is the only one present the value of z is in agreement with most recent calculations. [S1063-651X(96)07011-0]

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The dynamical critical exponent z of the two-dimensional Ising model has been studied extensively in the past. Many statistical mechanics techniques have been employed to determine this exponent. Among the methods used in the evaluation of z, we point out the Monte Carlo simulations [1–4], high-temperature series expansion [5,6], damage spreading [7,8], renormalization-group calculations [9,10], and ϵ expansion [11]. The value found for z in each of these calculations is in the range between 1.80 and 2.70, but most of them are around the value $z=2.10$.

In this work we have considered a two-dimensional ferromagnetic Ising model, in which the system is in contact with a heat bath at temperature T and is subject to an external flux of energy. These processes can be simulated by two competing dynamics: the contact with the heat bath is taken into account by the single spin-flip Glauber kinetics [12] and the flux of energy into the system is simulated by a process of the Kawasaki type [13], where we exchange nearest-neighbor spins, which preserves the order parameter of the model. In our case, we consider only the exchange of spins that favors an increase in the energy of the system. Therefore, this kind of Kawasaki process is not the usual relaxational one.

A very interesting feature about the competition between the Glauber and Kawasaki processes is the emergence of the phenomenon of self-organization [14]. It was shown in Ref. [14] that, within the dynamical pair approximation and for a two-dimensional square lattice, the system goes continuously from the ferromagnetic to the paramagnetic state as we increase the flux of energy. If we further increase this flux, the system self-organizes into an antiferromagnetic phase. We would like to point out that the pair approximation gives no self-organization when the exchange coupling between the nearest-neighbor spins is of the antiferromagnetic type [15]. In this case, the two-dimensional calculations show that the antiferromagnetic order is destroyed by a small input of energy into the system.

We have recently shown [16] that Monte Carlo simulation performed on the two-dimensional version of the ferromagnetic system maintains the picture of a self-organization phenomenon. Nevertheless, the antiferromagnetic phase appears only when the Kawasaki process is the dominant one. As we will show below, for each value of the competition parameter between the Glauber and Kawasaki dynamics, we calculate the dynamical critical exponent z along the critical line separating the ferromagnetic and paramagnetic stationary phases. Our approach to evaluate z involves the nonlinear response of the ferromagnetic order parameter of the system, prepared at its ground state, when it is left at its nonequilibrium stationary critical temperature. We also take advantage of the fact that, for large values of linear dimension L of the lattice, the relaxation of the ferromagnetic order parameter does not depend on the size L. In fact, this procedure was applied before [17] to find the dynamical tricritical exponent z_t of the two-dimensional spin-1 Ising model with single-ion anisotropy.

Here we consider a ferromagnetic Ising model on a square lattice with N lattice sites. The state of the system is represented by $\sigma=(\sigma_1,\sigma_2,\ \ldots\ ,\sigma_N)$, where the spin variable assumes the values $\sigma_i=\pm 1$. The energy of the system in the state σ is given by

$$E(\sigma)=-J\sum_{i,j}\ \sigma_i\sigma_j\,, \tag{1}$$

where we consider in the summation only pairs of spins that are nearest neighbors and $J>0$. Let $P(\sigma,t)$ be the probability of finding the system in the state σ at time t. The evolution of $P(\sigma,t)$ is given by the master equation

$$\frac{dP(\sigma,t)}{dt}=\sum_{\sigma'}\ [P(\sigma',t)W(\sigma',\sigma)-P(\sigma,t)W(\sigma,\sigma')], \tag{2}$$

where $W(\sigma',\sigma)$ gives the probability, per unit time, for the transition from the state σ' to state σ. The two competing processes that define the complete dynamical evolution of the system can be written as

$$W(\sigma',\sigma)=pW_G(\sigma',\sigma)+(1-p)W_K(\sigma',\sigma). \tag{3}$$

In the above equation

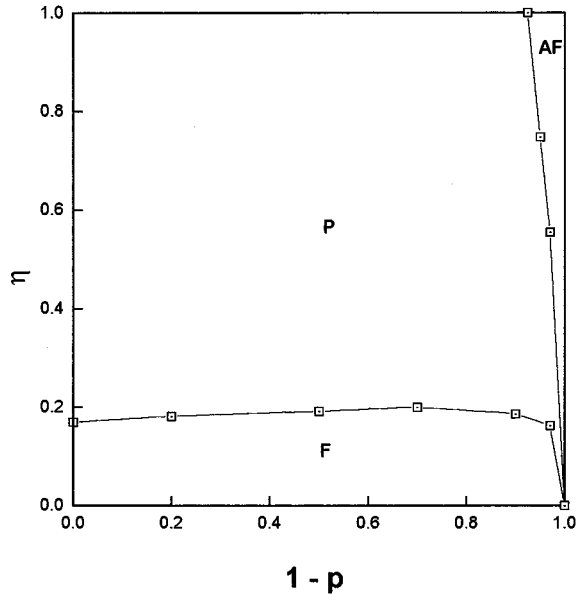

FIG. 1. Phase diagram of the two-dimensional kinetic ferromagnetic Ising model with competing Glauber (probability p) and Kawasaki (probability $1-p$) dynamics. The parameter η is given by $\eta=\exp(-J/\kappa_B T)$. The system exhibits the paramagnetic (P), ferromagnetic (F), and antiferromagnetic (AF) phases. The broken lines serve as a guide to the eyes.

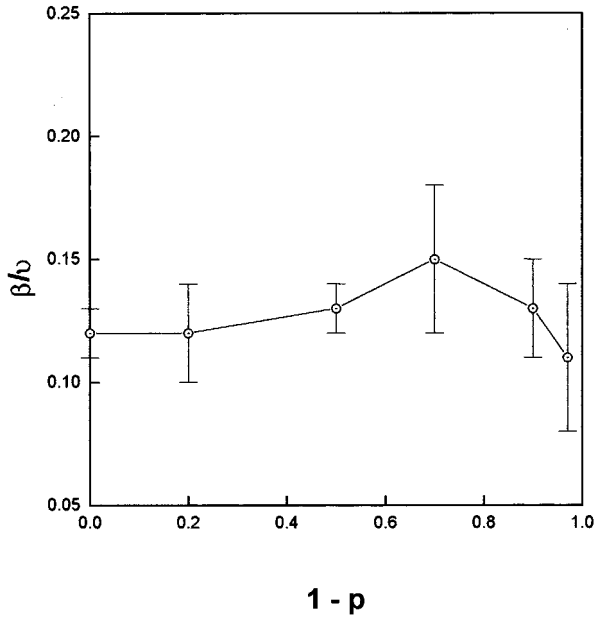

FIG. 2. Stationary values of the ratio β/ν as a function of $1-p$ at the transition point between the ferromagnetic and paramagnetic phases. The error bars give the accuracy of our Monte Carlo data points. We see that our estimated values of this ratio oscillate around the exact equilibrium value 1/8.

$$W_G(\sigma',\sigma)=\sum_{i=1}^{N}\delta_{\sigma_1',\sigma_1}\delta_{\sigma_2',\sigma_2},\ \ldots\ ,\delta_{\sigma_i'-\sigma_i},\ \ldots\ ,\times\delta_{\sigma_N',\sigma_N}w_i(\sigma) \tag{4}$$

is the single spin-flip Glauber process, which simulates the contact of our system with the heat bath at absolute temperature T and

$$W_K(\sigma',\sigma)=\sum_{i,j}\delta_{\sigma_1',\sigma_1}\delta_{\sigma_2',\sigma_2},\ldots,\delta_{\sigma_j',\sigma_j},\ldots,\times\delta_{\sigma_j',\sigma_i},\ldots,\delta_{\sigma_N',\sigma_N}w_{ij}(\sigma) \tag{5}$$

is the two-spin exchange Kawasaki process, which simulates the flux of energy into the system. In these equations $w_i(\sigma)$and $w_{ij}(\sigma)$ are, respectively, the probability, per unit time, of flipping spin i and the probability, per unit time, of exchanging two nearest-neighbor spins i and j. The $w_i(\sigma)$ and $w_{ij}(\sigma)$ are defined by

$$w_i(\sigma)=\min\left[1,\ \exp\left(-\frac{\Delta E_i}{\kappa_B T}\right)\right], \tag{6}$$

and

$$w_{ij}(\sigma)=\begin{cases}0 & \text{for } \Delta E_{ij}\leq 0\\ 1 & \text{for } \Delta E_{ij}>0,\end{cases} \tag{7}$$

where ΔE_i is the change in energy after flipping spin i and ΔE_{ij} is the change in energy after exchanging the neighboring spins i and j. Therefore, Eq. (7) shows that the effect of the Kawasaki dynamics is to favor antiferromagnetic bonding between neighboring spins, which increases the internal energy of the ferromagnetic system.

In order to obtain the phase diagram of the model we have performed Monte Carlo simulations on a square lattice with $L\times L=N$ sites, with the values of L ranging from $L=6$ up to 80. In all of our simulations we have used periodic boundary conditions. Also, we have started the simulations with different initial states in order to guarantee that the final stationary states we use in our calculations are the correct ones. For a given temperature T and a chosen value of the probability p, we choose at random a spin i from a given initial configuration. Then we generate a random number ξ_1 between zero and unity. If $\xi_1\leq p$ we choose to perform the Glauber process; in this process, we calculate the value of $w_i(\sigma)$. We again generate another random number ξ_2: if $\xi_2\leq w_i(\sigma)$, we flip spin i; otherwise we do not. If $\xi_1>p$ we go over the Kawasaki process. We generate another random number ξ_3 in order to select one of the four nearest neighbors of the spin i, say j. Then we find the value of w_{ij} and we exchange the selected spins only if $w_{ij}=1$. We note that after $10^4\times N$ Monte Carlo steps the stationary regime was established for all lattice sizes we consider. One Monte Carlo step equals N spin flips or exchanges of spins trials. In order to estimate the quantities of interest, we have used 5×10^4 Monte Carlo steps to calculate the averages for any lattice size.

The critical temperature for each value of p was determined by plotting the reduced fourth-order cumulant [18] as a function of temperature T, for several values of L. The resulting phase diagram [16] in the plane $\eta=\exp(-J/\kappa_B T)$versus $1-p$ is shown in Fig. 1. As we will show below, the estimates of the dynamical critical exponent z as a function of p depends on the previous determination of ratio β/ν, which is the scaling exponent for the magnetization in the neighborhood of the stationary critical point [19].

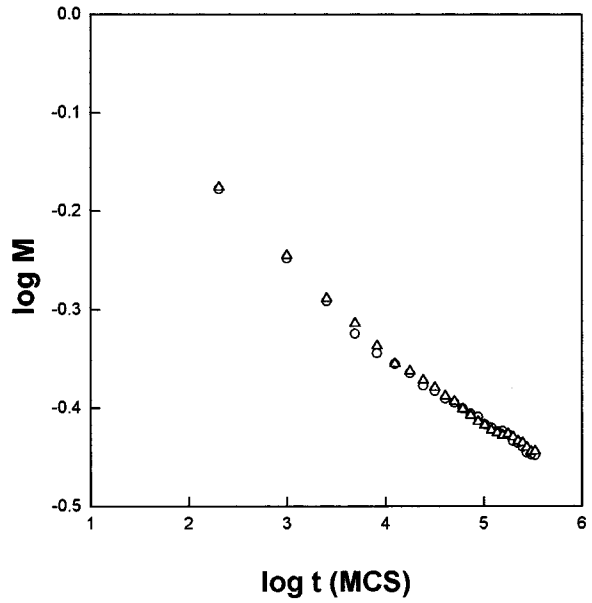

FIG. 3. Decay of magnetization as a function of time, for $p=0.5$, measured in Monte Carlo steps (MCS). Here we used natural logarithms. Measurements were made every 10 MCS, between 10 and 320 MCS. The lattice sizes are (160×160) (small circles) and (320×320) (small triangles).

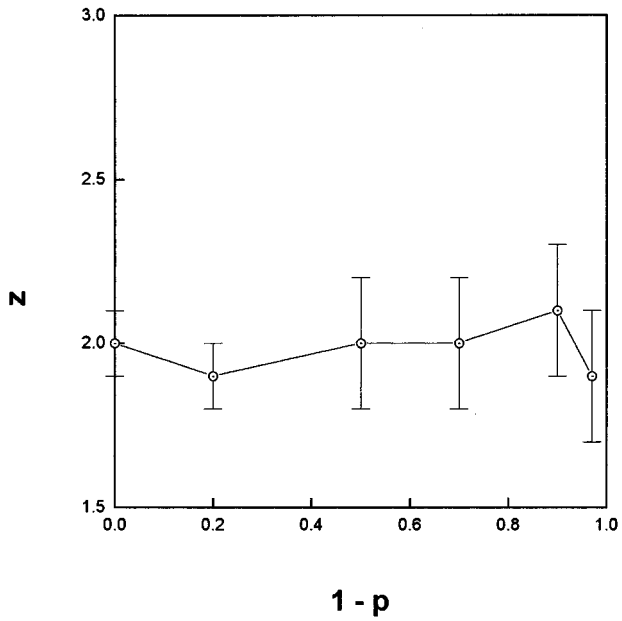

FIG. 4. Dynamical critical exponent z as a function of $1-p$ at the transition point between ferromagnetic and paramagnetic phases. The estimated values of z fluctuate around the value 2.0.

The magnetization is defined by

$$M_L=\langle|m|\rangle, \tag{8}$$

where $m=1/N\Sigma_{i=1}^{N}\sigma_i$, and it satisfies the scaling relation

$$M_L(T)=L^{-\beta/\nu}M_0(L^{1/\nu}\epsilon), \tag{9}$$

where $\epsilon=(T-T_c)/T_c$, T_c being the critical temperature for each value of p.

In Fig. 2 we exhibit the results we have found [16] for β/ν for several values of p. The exact value of this exponent for the equilibrium Ising model is well known and is given by $\beta/\nu=1/8$. As we can see, our estimated values for β/ν, depicted in Fig. 2, are in accordance with the corresponding values at equilibrium. This result gives support to the idea that the equilibrium and nonequilibrium Ising models, which exhibit up-down symmetry, belong to the same class of universality [20]. Recently, Bassler and Schmittmann [21] have extended these arguments about universality including also nonequilibrium two-state systems that do not respect the up-down symmetry of the equilibrium Ising model. A discussion concerning universality classes on driven diffusive systems, which settle into nonequilibrium steady states, is the subject of a review by Schmittmann and Zia [22].

Following Suzuki [23], the dynamic finite-size scaling theory asserts that the magnetization of a system of linear size L, at its critical point, evolves in time according to the scaling relation given by

$$M(t,L)=L^{-\beta/\nu}f(L^{-z}t). \tag{10}$$

Here we are interested in the determination of z. If we consider very large lattices, it is expected that the magnetization does not depend on the lattice size. Then it is easy to see that $M(t,L)$ can be written as

$$M(t,L)=At^{-\beta/\nu z}, \tag{11}$$

where A is a constant that does not depend on L. Equation (11) is valid only for large values of L. Then, taking into account Eq. (11), we can evaluate the exponent z, after a log-log plot of $M(t,L)$ versus t, for a fixed lattice size L, once we know the value of ratio β/ν, given in Fig. 2.

The Monte Carlo method was used again to follow the evolution of the magnetization in time for the competing model we are studying. First of all, we select a given value of competition parameter p. For this value of p we read in Fig. 1 its respective stationary critical temperature corresponding to the transition between the ferromagnetic and paramagnetic phases. After we prepare the system to be in its ground state, it evolves in time, measured in Monte Carlo steps per spin, and we recorded the magnetization at each of the ten Monte Carlo steps. The details of Monte Carlo simulation were given above.

In Fig. 3 we exhibit a log-log plot of $M(t)$ versus t for two values of the lattice size L, i.e., $L=160$ and 320, and for the selected value $p=0.5$. We can see that the decay of $M(t)$ is almost independent of L, which allow us to use Eq. (11) to evaluate the critical exponent z. In these calculations we have used 100 and 50 samples for the small and large lattices, respectively. We have also followed the decay of magnetization up to 320 Monte Carlo steps. If we discard the first 50 Monte Carlo steps, we can fit our data points to a straight line and we obtain from its slope the value $\beta/\nu z=0.065\pm0.001$. We have discarded the initial points of simulation because we want to put the system in its second regime, where a power-law decay of the magnetization is expected

[24]. Taking into account the value we have obtained for $\beta/\nu z$ and remembering from Fig. 2 that $\beta/\nu = 0.13 \pm 0.01$ if $p = 0.5$, we can estimate the value of the critical exponent z as being equal to 2.0 ± 0.2.

Finally, in Fig. 4 we exhibit a plot of the exponent z as a function of $1-p$. For all these values of p we have used lattices of size $L = 320$ and runs up to 320 Monte Carlo steps. As we can see, the values of the dynamical critical exponent fluctuate around $z = 2$.

This indicates that the underlying symmetries of this model are not affected by the flux of energy. We would like to point out that, as in our simulations the magnitude of our estimated errors can be as large as 0.2, the above assumption cannot be taken as a rigorous statement. But, we expect that, for whatever value of the rate of energy pumping into the system, the critical exponent z must remain the same because the intensity of the flux of energy cannot change the class of universality of this competing model. For the special case where $p = 1$, that is, when the system satisfies the detailed balance condition, we have found that $z = 2.0 \pm 0.1$. This value is in agreement with most calculations employing Monte Carlo methods (see Ref. [4] and references therein).

Although we do not present the detailed calculations concerning the continuous transition between the paramagnetic and antiferromagnetic phases, the critical temperature and the dynamical critical exponent can be obtained in a similar manner as we have done for the ferromagnetic to paramagnetic transition. For instance, if $p = 0.03$, we have found that $z = 2.0 \pm 0.2$.

In conclusion, we have calculated the dynamical critical exponent z for the nonequilibrium two-dimensional Ising model. The system is in contact with a heat bath at fixed temperature and subject to a continuous flux of energy. We have performed Monte Carlo simulations and used finite-size scaling relations to obtain the value of z at the stationary phase boundary between ordered (ferromagnetic or antiferromagnetic) and disordered paramagnetic phases. We have shown that the values found for z are independent of the magnitude of the flux of energy into the system. This type of universal behavior of z for the nonequilibrium Ising model is expected for this two-state Ising model if the flux of energy does not destroy the underlying symmetries of this system.

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[1] D. P. Landau, S. Tang, and S. Wansleben, J. Phys. C **8**, 1525 (1988).
[2] S. Tang and D. P. Landau, Phys. Rev. B **36**, 567 (1987).
[3] N. Jan, L. L. Moseley, and D. Stauffer, J. Stat. Phys. **33**, 1 (1988).
[4] M.-D. Lacasse, J. Viñals, and M. Grant, Phys. Rev. B **47**, 5646 (1993).
[5] J. Rogiers and J. O. Indekeu, Phys. Rev. B **41**, 6998 (1990).
[6] J. Wang, Phys. Rev. B **47**, 869 (1993).
[7] O. F. de Alcântara Bonfim, Europhys. Lett. **4**, 373 (1987).
[8] P. H. Poole and N. Jan, J. Phys. A **23**, L453 (1990).
[9] Y. Achiam and J. M. Kosterlitz, Phys. Rev. Lett. **41**, 128 (1978).
[10] G. F. Mazenko and O. T. Valls, Phys. Rev. B **24**, 1419 (1981).
[11] B. I. Halperin and P. C. Hohenberg, Rev. Mod. Phys. **49**, 435 (1977).
[12] R. J. Glauber, J. Math. Phys. **4**, 294 (1963).
[13] K. Kawasaki, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic, London, 1972), Vol. 4.
[14] T. Tomé and M. J. de Oliveira, Phys. Rev. A **21**, 6643 (1989).
[15] B. C. S. Grandi and W. Figueiredo, Phys. Rev. B **50**, 12 595 (1994).
[16] B. C. S. Grandi and W. Figueiredo, Phys. Rev. E **53**, 5484 (1996).
[17] O. F. de Alcântara Bonfim, J. Stat. Phys. **48**, 919 (1987).
[18] V. Privman, *Finite-Size Scaling and Numerical Simulation of Statistical Systems* (World Scientific, Singapore, 1990).
[19] K. Binder, Z. Phys. B **43**, 119 (1981).
[20] G. Grinstein, C. Jayaprakash, and Yu He, Phys. Rev. Lett. **55**, 2527 (1985).
[21] K. E. Bassler and B. Schmittmann, Phys. Rev. Lett. **73**, 3343 (1994).
[22] B. Schmittmann and R. K. P. Zia, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic, New York, in press).
[23] M. Suzuki, Prog. Theor. Phys. **58**, 1142 (1977).
[24] Z. Li, L. Schülke, and B. Zheng, Phys. Rev. E **53**, 2940 (1996).